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Torsion indices of smooth projective varieties

This is a report on a joint work with Andre Chatzistimatiou. For a smooth projective variety X of dimension d over a field k, we consider the *torsion index* $\mathrm{Tor}_k X$ of X, this being the order (possibly infinite) of the diagonal in $\mathrm{CH}_d((X \setminus F_0) \times (X \setminus F^1))$, where F_0 is a sufficiently large dimension zero subset of X and F^1 is a sufficiently large codimension one subset of X (one takes F_0 and F^1 large enough so that the order of the diagonal remains constant under further enlargement). For X geometrically integral, we consider as well the *geometric torsion index* of X, $\mathrm{Tor}_{\bar{k}}(X_{\bar{k}})$ where $\bar{k}$ is the algebraic closure of k. The torsion index gives an upper bound for the exponent of unramified cohomology on X.

We consider the geometric torsion index of a complete intersection $X^{d_1,\ldots,d_r;n}$ of multi-degree $d_1 \geq \ldots \geq d_r \geq 2$ in $\mathbb{P}^{n+r}$ and the torsion index of a generic complete intersection in $\mathbb{P}^{n+r}$. The geometric construction of Roitman gives the upper bound

$$\mathrm{Tor}_{\bar{k}}(X_{\bar{k}}^{d_1,\ldots,d_r;n}) \leq \prod_{i=1}^{r} d_i!$$

if $\sum_i d_i \leq n+r$. Using the method of Kollár, Voisin, Colliot-Thélène-Pirutka and Totaro, improved by this using the de Rham-Witt complex, we give a lower bound: Let $d_1 \geq \ldots \geq d_r \geq 2$ and $n \geq 3$ be integers such that $\sum_i d_i \leq n + r$. Let p be a prime number and let $m \geq 1$ be an integer such that

$$d_1 \geq p^m \cdot \left\lceil \frac{n+r+1-\sum_{i=2}^{r} d_i}{p^m+1} \right\rceil \qquad (0.1)$$

Then $p^m | \mathrm{Tor}(X)$ for all very general $X = X_{d_1,\ldots,d_r} \subset \mathbb{P}^{n+r}$.

Finally, we show that for the generic $X^{d_1,\ldots,d_r;n}$ in $\mathbb{P}^{n+r}$ (ie, the complete intersection defined by homogeneous equations with coefficients being independent variables $\ldots, \eta_i, \ldots$), we have the lower bound

$$\prod_{i=1}^{r} d_i!^* | \mathrm{Tor}_{k(\eta)} X^{d_1,\ldots,d_r;n}$$

Here, for a positive integer d, $d!^*$ is the least common multiple of the numbers $1, 2, \ldots, d$. For example, the generic cubic hypersurface in $\mathbb{P}^{n+1}$ with $n \geq 2$ has torsion index equal to six, while the generic quartic hypersurface in $\mathbb{P}^{n+1}$ with $n \geq 3$ has torsion index either 12 or 24.